The lepton flavor violating decays $Z \to l_i l_j$ in the simplest little Higgs model

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Abstract

In the simplest little Higgs model the new flavor-changing interactions between heavy neutrinos and the Standard Model leptons can generate contributions to some lepton flavor violating decays of Z-boson at one-loop level, such as $Z \to \tau^{\pm}\mu^{\mp}$, $Z \to \tau^{\pm}e^{\mp}$, and $Z \to \mu^{\pm}e^{\mp}$. We examine the decay modes, and find that the branching ratios can reach 10^{-7} for the three decays, which should be accessible at the Giga$Z$ option of the ILC.

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I. INTRODUCTION

Little Higgs theory [1] has been proposed as an interesting solution to the hierarchy problem. So far various realizations of the little Higgs symmetry structure have been proposed [2–5], which can be categorized generally into two classes [6]. One class use the product group, represented by the littlest Higgs model [3], in which the SM $SU(2)_L$ gauge group is from the diagonal breaking of two (or more) gauge groups. The other class use the simple group, represented by the simplest little Higgs model (SLHM) [4], in which a single larger gauge group is broken down to the SM $SU(2)_L$. The flavor sector of little Higgs models based on product groups, notably the littlest Higgs model with T-parity (LHT) [5], has been extensively studied [7]. Recently, some attentions have been paid to the flavor sector of SLHM [8–10].

The lepton flavor violating (LFV) decays of Z-boson can be a sensitive probe for new physics because they are extremely suppressed in the SM but can be greatly enhanced in new physics models [11–13]. The experimental limits obtained at LEP [14] are

$$\begin{aligned} BR(Z \to \tau^\pm \mu^\mp) &< 1.2 \times 10^{-5}, \\ BR(Z \to \tau^\pm e^\mp) &< 9.8 \times 10^{-6}, \\ BR(Z \to \mu^\pm e^\mp) &< 1.7 \times 10^{-6}. \end{aligned} \tag{1}$$

The next generation Z factory can be realized in the Giga$Z$ option of the International Linear Collider (ILC) [15]. About 2×10^9 Z events can be generated in an operational year of $10^7 s$ of Giga$Z$. Thus the expected sensitivity of Giga$Z$ to the LFV decays of Z-boson could reach [16]

$$\begin{aligned} BR(Z \to \tau^\pm \mu^\mp) &\sim \kappa \times 2.2 \times 10^{-8}, \\ BR(Z \to \tau^\pm e^\mp) &\sim \kappa \times 6.5 \times 10^{-8}, \\ BR(Z \to \mu^\pm e^\mp) &\sim 2.0 \times 10^{-9}, \end{aligned} \tag{2}$$

with the factor κ ranging from 0.2 to 1.0. Therefore, Giga$Z$ can offer an important opportunity to probe the new physics via the LFV decays of Z-boson.

The SLHM predicts the existence of heavy neutrinos, which have flavor-changing couplings with the SM leptons mediated respectively by the SM gauge boson $W^\pm$ and the new heavy gauge boson $X^\pm$. These couplings can give great contributions to Z-boson decays

$Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$, and $Z \to \mu^\pm e^\mp$ at one-loop level. In this paper, we will calculate the branching ratios of these decay modes, and compare the results with the sensitivity of Giga$Z$ and the present experimental bounds, respectively.

This work is organized as follows. In Sec. II we recapitulate the SLHM. In Sec. III we study respectively the decays $Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$ and $Z \to \mu^\pm e^\mp$. Finally, we give our conclusion in Sec. IV.

II. SIMPLEST LITTLE HIGGS MODEL

The SLHM is based on $[SU(3) \times U(1)_X]^2$ global symmetry [4]. The gauge symmetry $SU(3) \times U(1)_X$ is broken down to the SM electroweak gauge group by two copies of scalar fields Φ_1 and Φ_2, which are triplets under the $SU(3)$ with aligned VEVs f_1 and f_2. The uneaten five pseudo-Goldstone bosons can be parameterized as

$$\Phi_1 = e^{i\, t_\beta \Theta} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \Phi_2 = e^{-\frac{i}{t_\beta}\Theta} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}, \tag{3}$$

where

$$\Theta = \frac{1}{f} \left[\begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & H \\ H^\dagger & 0 \end{pmatrix} + \frac{\eta}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right], \tag{4}$$

$f = \sqrt{f_1^2 + f_2^2}$ and $t_\beta \equiv \tan\beta = f_2/f_1$. Under the $SU(2)_L$ SM gauge group, η is a real scalar, while H transforms as a doublet and can be identified as the SM Higgs doublet. The kinetic term in the non-linear sigma model is

$$\mathcal{L}_\Phi = \sum_{j=1,2} \left| \left(\partial_\mu + igA_\mu^a T^a - i\frac{g_x}{3} B_\mu^x \right) \Phi_j \right|^2, \tag{5}$$

where $g_x = gt_W/\sqrt{1 - t_W^2/3}$, and $t_W = \tan\theta_W$ with θ_W being the electroweak mixing angle. As Φ_1 and Φ_2 develop their VEVs, the new heavy gauge bosons Z', Y^0, $Y^{0\dagger}$ and $X^\pm$ get

their masses after eating five Goldstone bosons,

$$\begin{aligned} M_X &= \frac{gf}{\sqrt{2}}\left(1-\frac{v^2}{4f^2}\right), \\ M_{Z'} &= \frac{\sqrt{2}gf}{\sqrt{3-t_W^2}}\left(1-\frac{3-t_W^2}{c_W^2}\frac{v^2}{16f^2}\right), \\ M_Y &= \frac{gf}{\sqrt{2}}. \end{aligned} \tag{6}$$

The gauged $SU(3)$ symmetry promotes the SM fermion doublets into $SU(3)$ triplets. For each generation of lepton, a heavy neutrino is added, whose mass is

$$m_{N_i} = f s_\beta \lambda_N^i. \tag{7}$$

Where $i = 1, 2, 3$ is the generation index and λ_N^i is the Yukawa coupling constant.

After the EWSB the light and the heavy neutrino of the same family have the mixing, which is parameterized by $\delta_v = -\frac{v}{\sqrt{2}ft_\beta}$. The mixing angel δ_v is experimentally constrained to be small [17], and taken as a typical upper limit $\delta_v < 0.05$ following the ref. [8]. Besides, there is family mixing as long as the Yukawa matrix of heavy neutrinos and that of leptons are not aligned. This can induce the lepton flavor-changing interactions of charged currents proportional to $V_\ell^{ij}\bar{N}_{Li}\gamma^\mu X^{+\mu}\ell_{Lj}$ and $\delta_v V_\ell^{ij}\bar{N}_{Li}\gamma^\mu W^{+\mu}\ell_{Lj}$, where V_ℓ^{ij} is the mixing matrix [6, 8, 9].

III. THE LFV DECAYS $Z \to \tau^\pm\mu^\mp$, $Z \to \tau^\pm e^\mp$ AND $Z \to \mu^\pm e^\mp$

In the SLHM, the Feynman diagrams for $Z \to \mu^\pm e^\mp$ can be depicted by the Fig. 1, and the diagrams for $Z \to \tau^\pm\mu^\mp$, $Z \to \tau^\pm e^\mp$ are same as Fig.1, but replacing μ and e with the corresponding final particles. For the 't Hooft-Feynman gauge, the flavor-changing interactions between the heavy neutrino and lepton, mediated by the gauge bosons (Goldstone bosons) $X^\pm$ ($x^\pm$) and $W^\pm$ ($\phi^\pm$), can contribute to these decays. The relevant Feynman rules can be found in [8].

The calculations of the loop diagrams in Fig. 1 are straightforward. Each loop diagram is composed of some scalar loop functions [18] which are calculated by using LoopTools [19]. The analytic expressions from our calculation are presented in Appendix A.

The SM input parameters relevant in our study are taken as ref. [20]. The free SLHM parameters involved are f, t_β, the heavy neutrino mass m_{N_i} ($i = 1, 2, 3$), and the mixing

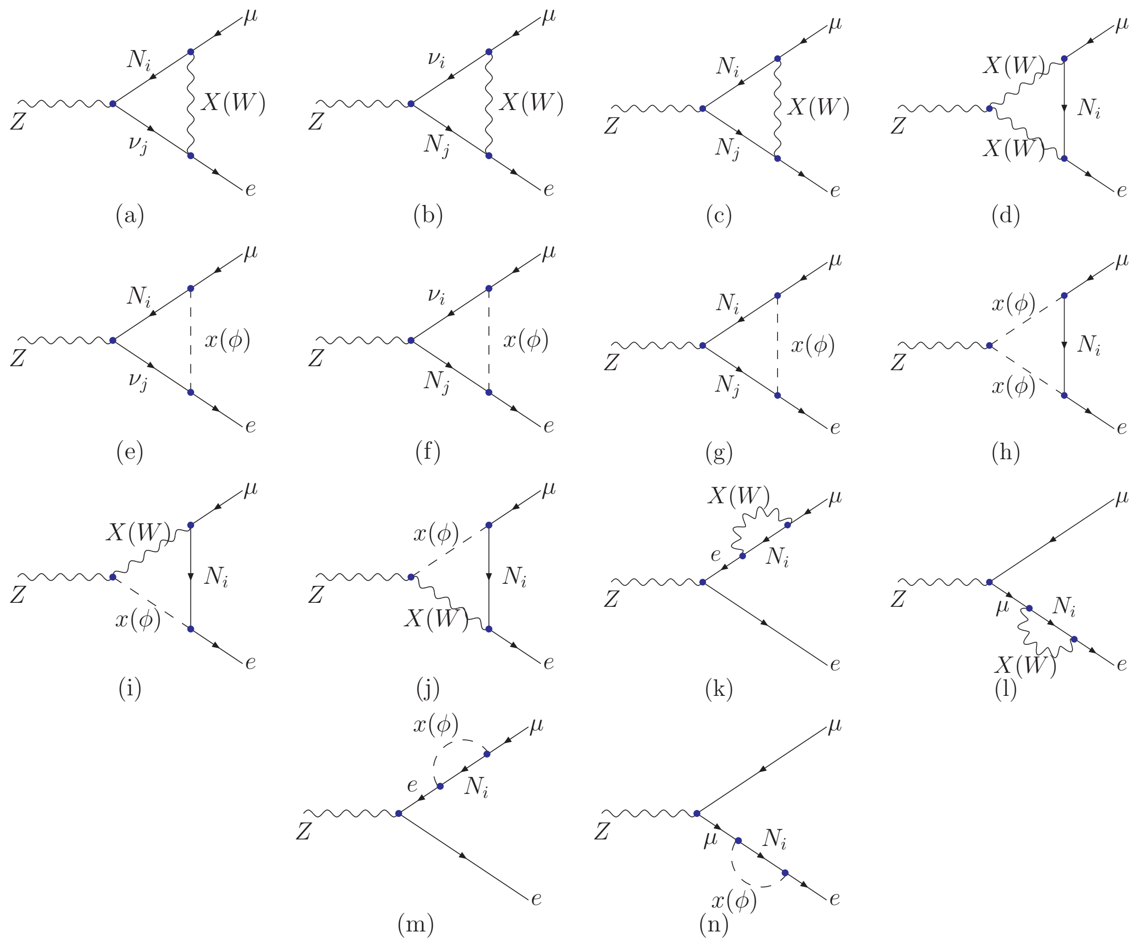

FIG. 1: Feynman diagrams for $Z \to \mu^+ e^-$ in the SLHM.

matrix V_ℓ which can be parameterized with standard form. To simply our calculations, we take the parameters [21]

$$s_{12} = \sqrt{0.3}, \quad s_{13} = \sqrt{0.03}, \quad s_{23} = \frac{1}{\sqrt{2}}, \quad \delta_{13} = 65^\circ, \tag{8}$$

which is consistent with the experimental constraints on the PMNS matrix [22], and δ_{13} is taken to be equal to the CKM phase. To satisfy the present experimental bounds of $Br(\mu \to e\gamma)$ and $Br(\mu \to eee)$, the mass splitting of the first and the second heavy neutrinos must be very small [8]. So in this paper we will take $m_{N_1} = m_{N_2} = m_1 = 400$ GeV and $m_{N_3} = m_3$ in the range of 500 GeV-3000 GeV. Ref. [4] shows that the LEP-II data requires $f > 2$ TeV. In our numerical calculation we will take several values of t_β for $f = 2$ TeV, $f = 4$ TeV and $f = 5.6$ TeV, respectively.

In Fig. 2, Fig. 3 and Fig. 4, we plot the decay branching ratios of $Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$

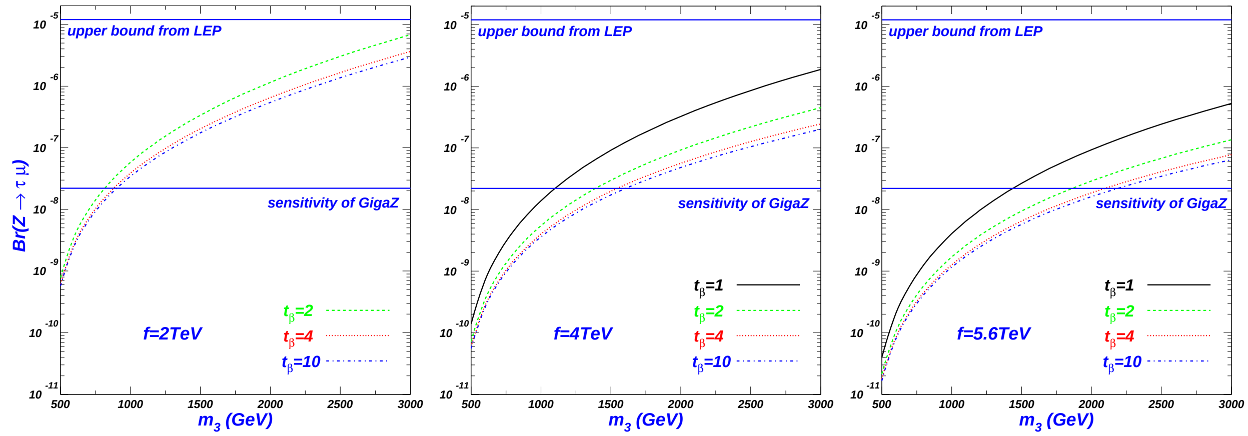

FIG. 2: The branching ratios of $Z \to \tau^\pm\mu^\mp$ versus m_3.

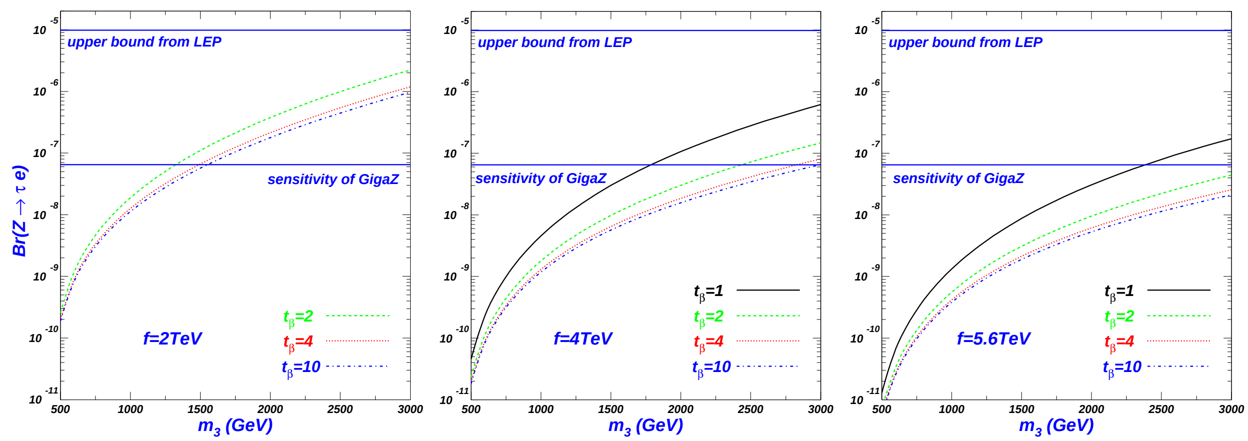

FIG. 3: The branching ratios of $Z \to \tau^\pm e^\mp$ versus m_3.

and $Z \to \mu^\pm e^\mp$ versus m_3 for $f = 2$ TeV, $f = 4$ TeV and $f = 5.6$ TeV, respectively. We find that the branching ratios increase with the mass of the third generation heavy neutrino. The reason is that the decays are enhanced by the large mass splitting $m_3 - m_1$, which increases as m_3 gets large since we have fixed the value of m_1. Besides, the branching ratios drop as the scale f or t_β get large, and the reason is that the lepton flavor-changing couplings $\bar{N}_{Li}\gamma^\mu W^{+\mu}\ell_{Lj}$ and $\bar{N}_i\phi^+\ell_j$ are proportional to $\delta_v = -\frac{v}{\sqrt{2}ft_\beta}$.

Fig. 2, Fig. 3 and Fig. 4 show the branching ratios of $Z \to \tau^\pm\mu^\mp$, $Z \to \tau^\pm e^\mp$ and $Z \to \mu^\pm e^\mp$ are below the present experimental upper bounds, respectively. However, the ratios can be enhanced to reach the sensitivity of the Giga$Z$. For $f = 2$ TeV, $t_\beta = 4$ and $m_3 = 2$ TeV, the branching ratios can reach 10^{-7} for $Z \to \tau^\pm\mu^\mp$, $Z \to \tau^\pm e^\mp$ and $Z \to \mu^\pm e^\mp$, which exceed much the sensitivity of Giga$Z$. In the LHT, all the three ratios can reach 10^{-6} [12]. Therefore, the LFV decays of Z-boson may be accessible at Giga$Z$, and thus may serve as a probe of the little Higgs models.

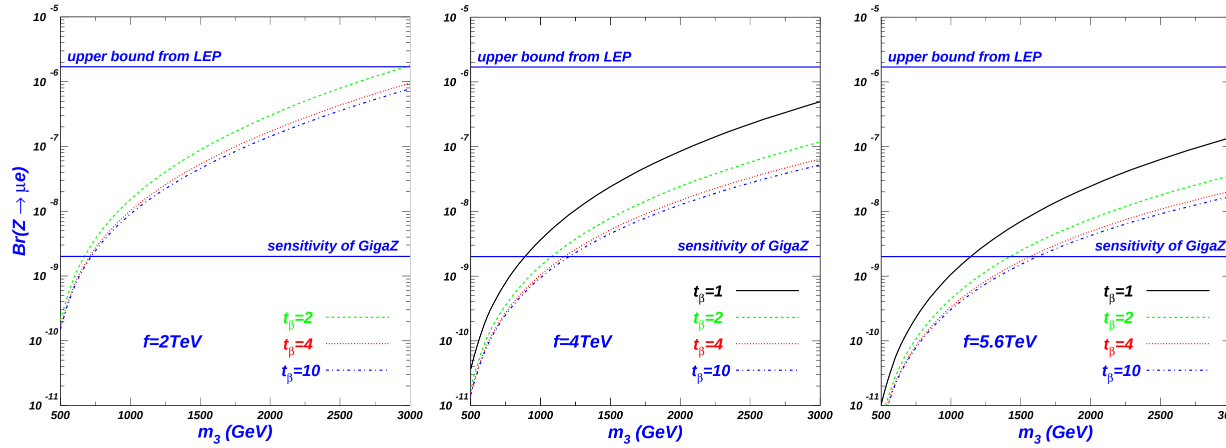

FIG. 4: The branching ratios of $Z \to \mu^\pm e^\mp$ versus m_3.

IV. CONCLUSION

In the framework of the simplest little Higgs model, we studied the LFV decays $Z \to \tau^\pm\mu^\mp$, $Z \to \tau^\pm e^\mp$ and $Z \to \mu^\pm e^\mp$. In the parameter space allowed by current experiments, the branching ratios of the three decays can exceed respectively much the sensitivity of Giga$Z$, which should be accessible at the Giga$Z$ option of the ILC. Therefore, the measurement of these rare decays at the Giga$Z$ may serve as a probe of the simplest little Higgs model.

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Appendix A: The effective coupling of $Z\mu^+e^-$

Here we take the effective coupling of $Z\mu^+e^-$ for example. The other two couplings $Z\tau^+\mu^-$ and $Z\tau^+e^-$ can be obtained via some corresponding replacement of the analytic

expressions for $Z\mu^+e^-$. The effective coupling of $Z\mu^+e^-$ is given by

$$\begin{aligned}\Gamma^{\alpha}_{Z\mu e} \;=\;& \Gamma^{\alpha}_{VF_1F_2}\big[X(W),N_i,\nu_j\big]+\Gamma^{\alpha}_{VF_1F_2}\big[X(W),\nu_i,N_j\big]+\Gamma^{\alpha}_{VF_1F_2}\big[X(W),N_i,N_j\big]\\ &+\Gamma^{\alpha}_{FVV}\big[N_i,X(W),X(W)\big]+\Gamma^{\alpha}_{SF_1F_2}\big[x(\phi),N_i,\nu_j\big]+\Gamma^{\alpha}_{SF_1F_2}\big[x(\phi),\nu_i,N_j\big]\\ &+\Gamma^{\alpha}_{SF_1F_2}\big[x(\phi),N_i,N_j\big]+\Gamma^{\alpha}_{FSS}\big[N_i,x(\phi),x(\phi)\big]+\Gamma^{\alpha}_{FVS}\big[N_i,X(W),x(\phi)\big]\\ &+\Gamma^{\alpha}_{FSV}\big[N_i,x(\phi),X(W)\big]+\Gamma^{\alpha}_{self(k)}\big[X(W),N_i\big]+\Gamma^{\alpha}_{self(l)}\big[X(W),N_i\big]\\ &+\Gamma^{\alpha}_{self(m)}\big[x(\phi),N_i\big]+\Gamma^{\alpha}_{self(n)}\big[x(\phi),N_i\big],\end{aligned}\tag{A1}$$

where the particles in the square brackets represent the particles which contribute to the vertex, and $\Gamma^{\alpha}_{self(k-n)}$ correspond to the vertexes in Fig. $1(k-n)$. The self-energy and vertex contributions in the above equation are given by

$$\begin{aligned}\Gamma^{\alpha}_{VF_1F_2} \;=\;& \frac{i}{16\pi^2}\big[(d_1Z^f_RP_L+c_1Z^f_LP_R)(-2C_{\sigma\rho}\gamma^{\sigma}\gamma^{\alpha}\gamma^{\rho}-2\gamma^{\alpha})(c_2P_L+d_2P_R)\\ &-2(\not{q}_e+\not{q}_\mu)\gamma^{\alpha}C_\beta\gamma^{\beta}(c_1c_2Z^f_LP_L+d_1d_2Z^f_RP_R)+4m_{F_2}(c_2d_1Z^f_RP_L\\ &+c_1d_2Z^f_LP_R)C_\alpha+4m_{F_1}(c_2d_1Z^f_LP_L+c_1d_2Z^f_RP_R)C_\alpha+2m_{F_1}C_0(d_1Z^f_LP_L\\ &+c_1Z^f_RP_R)(2(q_e+q_\mu)^{\alpha}-m_{F_2}\gamma^{\alpha})(c_2P_L+d_2P_R)\big](q_e,q_\mu,m_{F1},m_V,m_{F2}),\end{aligned}\tag{A2}$$

$$\begin{aligned}\Gamma^{\alpha}_{FVV} \;=\;& \frac{ig_{VVV}}{16\pi^2}(d_1P_L+c_1P_R)\big\{-4C_{\alpha\beta}\gamma^{\beta}+\gamma^{\alpha}-2C_\beta\gamma^{\beta}(q_e+q_\mu)^{\alpha}+2(4m_F-2\not{q}_\mu)C_\alpha\\ &+(4m_F-2\not{q}_\mu)(q_e+q_\mu)^{\alpha}C_0-\big[C_{\sigma\rho}g^{\sigma\rho}-\frac{1}{2}\big]\gamma^{\alpha}-C_\beta\gamma^{\beta}(\not{q}_\mu+m_F)\gamma^{\alpha}-\not{p}_ZC_\beta\gamma^{\beta}\gamma^{\alpha}\\ &-\not{p}_Z(\not{q}_\mu+m_F)\gamma^{\alpha}C_0-\big[C_{\sigma\rho}g^{\sigma\rho}-\frac{1}{2}\big]\gamma^{\alpha}+C_\beta\gamma^{\alpha}\gamma^{\beta}(\not{p}_Z-\not{q}_e-\not{q}_\mu)-\gamma^{\alpha}(\not{q}_\mu+m_F)C_\beta\gamma^{\beta}\\ &+\gamma^{\alpha}(\not{q}_\mu+m_F)(\not{p}_Z-\not{q}_e-\not{q}_\mu)C_0\big\}(c_2P_L+d_2P_R)(q_\mu,q_e,m_V,m_F,m_V),\end{aligned}\tag{A3}$$

$$\begin{aligned}\Gamma^{\alpha}_{SF_1F_2} \;=\;& \frac{i}{16\pi^2}\big[C_{\sigma\rho}\gamma^{\sigma}\gamma^{\alpha}\gamma^{\rho}(a_2b_1Z^f_RP_L+a_1b_2Z^f_LP_R)+\frac{1}{2}\gamma^{\alpha}(a_2b_1Z^f_RP_L+a_1b_2Z^f_LP_R)\\ &+C_\beta\gamma^{\beta}\gamma^{\alpha}(a_1Z^f_LP_L+b_1Z^f_RP_R)(\not{q}_e+\not{q}_\mu+m_{F_2})(a_2P_L+b_2P_R)\\ &+m_{F_1}\gamma^{\alpha}(b_1b_2Z^f_LP_L+a_1a_2Z^f_RP_R)C_\beta\gamma^{\beta}+m_{F_1}\gamma^{\alpha}(b_1Z^f_LP_L\\ &+a_1Z^f_RP_R)(\not{q}_e+\not{q}_\mu+m_{F_2})(a_2P_L+b_2P_R)C_0\big](q_e,q_\mu,m_{F1},m_S,m_{F2}),\end{aligned}\tag{A4}$$

$$\Gamma^{\alpha}_{FSS} = -\frac{ig_{VSS}}{16\pi^2}\big\{-2C_{\alpha\beta}\gamma^{\beta}(a_2b_1P_L+a_1b_2P_R)-(q_e+q_\mu)^{\alpha}C_{\beta}\gamma^{\beta}(a_2b_1P_L+a_1b_2P_R) +\big[-2C_{\alpha}-(q_e+q_\mu)^{\alpha}C_0\big]\not{q}_\mu(a_2b_1P_L+a_1b_2P_R)+m_F\big[-2C_{\alpha} -(q_e+q_\mu)^{\alpha}C_0\big](a_1a_2P_L+b_1b_2P_R)\big\}(q_\mu,q_e,m_S,m_F,m_S), \quad \text{(A5)}$$

$$\Gamma^{\alpha}_{FVS} = -\frac{ig_{VVS}}{16\pi^2}\gamma^{\alpha}(c_1P_L+d_1P_R)\big[C_{\beta}\gamma^{\beta}+(\not{q}_\mu+m_F)C_0\big] \times(a_2P_L+b_2P_R)(q_\mu,q_e,m_S,m_F,m_V), \quad \text{(A6)}$$

$$\Gamma^{\alpha}_{FSV} = \frac{ig_{VVS}}{16\pi^2}(a_1P_L+b_1P_R)\big[C_{\beta}\gamma^{\beta}+(\not{q}_\mu+m_F)C_0\big]\gamma^{\alpha} \times(c_2P_L+d_2P_R)(q_\mu,q_e,m_V,m_F,m_S), \quad \text{(A7)}$$

$$\Gamma^{\alpha}_{self(k)} = -\frac{ig}{16\pi^2c_W(q_\mu^2-m_e^2)}\gamma^{\alpha}\big[(-\frac{1}{2}+s_W^2)P_L+s_W^2P_R\big](\not{q}_\mu+m_e)\big[(2B_{\beta}\gamma^{\beta}+(2B_0 -1)\not{q}_\mu)(c_1c_2P_L+d_1d_2P_R)-2m_F(2B_0-1)(c_2d_1P_L+c_1d_2P_R)\big](q_\mu,m_V,m_F), \quad \text{(A8)}$$

$$\Gamma^{\alpha}_{self(l)} = -\frac{ig}{16\pi^2c_W(p_e^2-m_\mu^2)}\big[(2B_{\beta}\gamma^{\beta}+(2B_0-1)\not{p}_e)(c_1c_2P_L+d_1d_2P_R)-2m_F(2B_0 -1)(c_2d_1P_L+c_1d_2P_R)\big](\not{p}_e+m_\mu)\gamma^{\alpha}\big[(-\frac{1}{2}+s_W^2)P_L+s_W^2P_R\big](p_e,m_V,m_F), \quad \text{(A9)}$$

$$\Gamma^{\alpha}_{self(m)} = \frac{ig}{16\pi^2c_W(q_\mu^2-m_e^2)}\gamma^{\alpha}\big[(-\frac{1}{2}+s_W^2)P_L+s_W^2P_R\big](\not{q}_\mu+m_e)\big[(B_{\beta}\gamma^{\beta} +\not{q}_\mu B_0)(a_2b_1P_L+a_1b_2P_R)+m_FB_0(a_1a_2P_L+b_1b_2P_R)\big](q_\mu,m_S,m_F), \quad \text{(A10)}$$

$$\Gamma^{\alpha}_{self(n)} = \frac{ig}{16\pi^2c_W(p_e^2-m_\mu^2)}\big[(B_{\beta}\gamma^{\beta}+\not{p}_eB_0)(a_2b_1P_L+a_1b_2P_R)+m_FB_0(a_1a_2P_L +b_1b_2P_R)\big](\not{p}_e+m_\mu)\gamma^{\alpha}\big[(-\frac{1}{2}+s_W^2)P_L+s_W^2P_R\big](p_e,m_S,m_F), \quad \text{(A11)}$$

where $q_\mu = -p_\mu$, $q_e = -p_e$ and $P_{L,R} = (1 \mp \gamma_5)/2$. The functions B and C are 2- and 3-point Feynman integrals [19], and their functional dependence is indicated in the bracket following them. The tensor loop functions can be expanded as the scalar functions [19]. In our calculation the contraction of Lorentz indices is performed numerically. The parameters appearing above are from

$$V\bar{e}f : i\gamma^{\mu}(c_1P_L+d_1P_R), \qquad V\bar{f}\mu : i\gamma^{\mu}(c_2P_L+d_2P_R),$$

$$S\bar{e}f : a_1P_L+b_1P_R, \qquad S\bar{f}\mu : a_2P_L+b_2P_R,$$

$$ZS^+S^- : ig_{VSS}(p^{\mu}_{S^+}-p^{\mu}_{S^-}), \qquad ZV^+S^- : g_{VVS}g^{\mu\nu},$$

$$Z_{\rho}V^+_{\mu}V^-_{\nu} : -ig_{VVV}[(p_{V^+}-p_{V^-})^{\rho}g^{\mu\nu}+(p_Z-p_{V^+})^{\nu}g^{\mu\rho}+(p_{V^-}-p_Z)^{\mu}g^{\nu\rho}],$$

$$Z\bar{f}_1f_2 : i\gamma^{\mu}(Z^f_LP_L+Z^f_RP_R),$$

where V represents gauge bosons and S represents scalar particles. These couplings represent the seven different classes of vertices involved in our calculation. In each class of vertices,

the parameters a_1, b_1, a_2, b_2, c_1, d_1, c_2, d_2, g_{VSS}, g_{VVS}, g_{VVV}, Z_L^f and Z_R^f take different values for different concrete coupling. The analytic expressions of these parameters can be found in [8].

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